How does Quantum Energy Teleportation manifest in a two-electron system

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***Abstract* -** I propose a Quantum Energy Teleportation (QET) protocol specifically for a system composed of two entangled electrons. This protocol allows for the transfer of energy between the two electrons by means of local operations and classical communication (LOCC), without violating any physical laws such as causality and local energy conservation. The process leverages the inherent quantum entanglement of the electron pair, enabling the teleportation of energy without any direct physical exchange of particles. By performing a projective measurement on one electron and transmitting the result via a classical channel, energy can be extracted from the other electron through a local unitary operation. This paper focuses on the theoretical analysis of QET in a minimal two-electron system, offering insights into the fundamental principles governing energy dynamics and the conditions necessary for successful energy teleportation

I. INTRODUCTION

Quantum entanglement is a fundamental feature of quantum mechanics that gives rise to several intriguing phenomena, including quantum teleportation (QT). In traditional QT, a quantum state can be transferred from one location to another using only local operations and classical communication (LOCC). However, this process does not inherently involve the teleportation of energy. For example, teleporting a spin-up state in a magnetic field does not transfer the associated energy but merely replicates the state at a distant location. The question arises: can energy itself be teleported using LOCC? Quantum mechanics allows this under specific conditions, which we explore through the protocol known as Quantum Energy Teleportation (QET).

In this paper, we investigate the QET protocol within a simplified system of **two entangled electrons**. We aim to understand how energy can be transferred between these two electrons without violating any physical laws such as causality or local energy conservation. The protocol begins with Alice performing a local measurement on her electron, which introduces energy into the system. The result of this measurement is then communicated to Bob, who performs a local operation on his electron, leading to the extraction of energy. By utilizing ground-state entanglement and localized excitations, this protocol demonstrates that energy transfer is possible even in minimal quantum systems like two-electron setups.

In contrast to more complex spin chains, where interactions extend over many particles, the two-electron system allows us to explore the fundamental principles of QET in a more controlled environment. This study provides a theoretical framework for understanding energy teleportation in the simplest possible entangled system, highlighting the role of quantum correlations and local operations in the process.

The concept of **Quantum Teleportation** (QT) was first introduced by Bennett et al. (1993), who demonstrated that an unknown quantum state could be transferred between two parties using shared entanglement and classical communication channels. This foundational work laid the groundwork for various quantum information protocols, but notably, QT did not involve the direct transfer of energy. The focus was instead on the replication of quantum information at a remote location, without involving the physical transport of particles. This fundamental idea of teleportation, though groundbreaking, left open the question of whether energy itself could be transferred in a similar manner, a challenge later addressed by Quantum Energy Teleportation (QET).

Building upon the concept of quantum state transfer, **Quantum Energy Teleportation (QET)** was first proposed by Hotta (2008), who introduced a protocol wherein energy could be teleported from one region to another using local operations and classical communication (LOCC). Hotta's work demonstrated that ground-state entanglement within a quantum system could serve as a resource for energy transfer, a result that defied classical intuition. The key insight of QET is that energy can be extracted from a subsystem (such as Bob’s electron) after another subsystem (such as Alice’s electron) has been perturbed via local measurement. This energy transfer occurs without the physical exchange of particles and instead relies on the underlying entanglement of the quantum state.

In the context of **spin chains**, Hotta’s original work analyzed the QET protocol within many-body quantum systems, where entanglement typically extends across multiple spins. Spin chains are well-known models in quantum information theory and condensed matter physics, used to study complex phenomena such as quantum phase transitions, quantum entanglement, and nonlocal correlations. These models provide a fertile ground for studying QET due to their inherent entanglement properties, with energy transfer mechanisms tied to the intricate web of correlations between spins. Subsequent works have expanded upon Hotta’s ideas, demonstrating that QET is not only theoretically sound but can also be extended to more complex quantum systems, including those exhibiting long-range interactions and topological properties.

The development of QET within **many-body systems** has led to significant insights into the interplay between quantum entanglement and energy dynamics. For instance, studies by various authors have explored QET in Heisenberg spin models, Bose-Hubbard systems, and harmonic oscillator networks, revealing the versatility of QET in different quantum mechanical contexts. These studies illustrate that energy teleportation can occur in a wide range of systems, each characterized by different types of entanglement structures and interactions. However, the complexity of these systems often masks the fundamental principles of QET, making it challenging to distill the underlying mechanisms that enable energy transfer.

Despite the success of QET in many-body systems, the simplest possible application of the protocol—namely, in a **two-electron system**—has received comparatively less attention in the literature. A two-electron system offers an ideal testing ground for understanding the core principles of QET in a minimal setting. In such systems, the entanglement is limited to only two particles, allowing researchers to isolate and analyze the energy dynamics without the complications introduced by multi-body interactions. This focus on minimal systems is inspired by foundational studies in quantum information theory, which often use two-particle models (such as Bell states or the Einstein-Podolsky-Rosen pair) to explore the nature of entanglement. By applying QET to a two-electron system, we can gain deeper insights into how energy is teleported and conserved in quantum systems, free from the distractions of more complex interactions.

II. METHODOLOGY

2. Background and Theoretical Framework

2.1 Quantum Entanglement

Quantum entanglement is a fundamental feature of quantum mechanics wherein the quantum states of two or more particles become interdependent, even when the particles are spatially separated. In a two-electron system, the entanglement manifests as a correlation between the spin states of the electrons. For example, consider the singlet Bell state, a maximally entangled state given by:

$$|\psi\rangle = \frac{1}{2}(|01\rangle - |10\rangle)$$

where $|0\rangle$ *and* $|1\rangle$ represent the spin-down and spin-up states of the electrons, respectively. When one electron's spin is measured, the state of the other electron is instantly determined, regardless of the distance between them. Entanglement is crucial for Quantum Energy Teleportation (QET), as it provides the nonlocal correlations required for energy transfer between Alice and Bob through local operations and classical communication (LOCC).

2.2 Quantum Teleportation (QT)

Quantum Teleportation (QT) is a well-known protocol in quantum information theory that allows for the transfer of an unknown quantum state between two parties using entanglement and classical communication. However, traditional QT does not transfer the energy associated with the quantum state but merely replicates the state at a distant location. For example, in QT, teleporting a spin-up state in a magnetic field does not transfer the excitation energy but only the information about the state.

In contrast, Quantum Energy Teleportation (QET) allows for the transfer of energy itself via LOCC. This occurs because the energy is stored nonlocally in the entangled state, allowing energy to be effectively transported from one location to another without physically moving particles.

2.3 Hamiltonian and Local Energy

In quantum mechanics, the Hamiltonian represents the total energy of a system, encompassing both kinetic and potential energy. In the context of a two-electron system, we model the interaction between the electrons' spins through a Hamiltonian of the form:

$$H = JS_A \cdot S_b,$$

where S_A *and* S_B are the spin operators of Alice's and Bob's electrons, respectively, and J is the coupling constant representing the strength of the interaction. The Hamiltonian plays a crucial role in determining the dynamics of energy exchange between the two electrons during the QET process.

Local energy density is also a key concept in QET. When Alice performs a local measurement on her electron, she introduces energy into the system. Through the QET protocol, a portion of this energy is teleported to Bob's electron, even though no physical particles are transferred.

2.4 Ground-State Entanglement and Local Energy Excitations

In QET, the ground-state entanglement of the two-electron system is central to the process of energy transfer. The ground state is the lowest energy state of the system and is typically an entangled state. For a two-electron system, this entangled ground state ensures that quantum correlations between Alice's and Bob's electrons are maintained, allowing for the teleportation of energy through local operations.

The QET protocol exploits local quantum fluctuations in the ground state. When Alice performs a projective measurement on her electron, the entanglement is disturbed, resulting in a localized excitation. This localized energy, introduced by Alice, is then teleported to Bob's electron through entanglement and classical communication, allowing Bob to extract the energy locally.

2.5 Quantum Energy Teleportation (QET) Protocol

The QET protocol can be broken down into three primary steps:

1. **Alice's Local Measurement**: Alice performs a projective measurement on her electron's spin state, introducing a localized excitation (energy) into the system.
2. **Classical Communication**: Alice sends the result of her measurement to Bob via a classical communication channel. This step is critical because it allows Bob to adjust his local operation based on Alice's measurement outcome.
3. **Bob's Local Operation**: Using the information received from Alice, Bob performs a local unitary operation on his electron. This operation allows Bob to extract energy from the system based on the quantum correlations between his electron and Alice's electron.

Through this sequence, energy is effectively teleported from Alice's electron to Bob's electron without violating any fundamental physical laws, including causality and local energy conservation. The teleportation of energy is enabled by the pre-existing entanglement between the electrons and the carefully coordinated local operations and classical communication.

2.6 Quantum Correlations and Two-Point Functions

Quantum correlations are quantified by two-point correlation functions, which describe the relationship between the states of Alice's and Bob's electrons. These correlation functions are essential for determining how the measurement outcome at Alice's site influences the local operation at Bob's site.

For a two-electron system, the correlation functions are defined as:

$$\xi = \langle \psi \mid \sigma_B H \sigma_B \mid \psi \rangle$$

$$\eta = \langle \psi \mid \sigma_A \sigma^{\cdot}{}_A \mid \psi \rangle,$$

where σ_A and σ_B are spin operators, and H is the Hamiltonian. These functions describe how the measurement and subsequent operation at Alice's site affect the energy extraction at Bob's site.

2.7 Local Energy Conservation

One of the most important principles in QET is local energy conservation. After Alice performs her measurement, energy is introduced locally at her site. The QET protocol ensures that Bob can only extract energy from the system that is accounted for by this initial energy input. The system conserves energy locally, meaning that any energy Bob extracts corresponds to an equal and opposite change in the local energy density near Bob's electron.

In QET, negative energy density regions may appear after Bob's operation. These regions ensure that the total energy remains conserved while allowing Bob to extract positive energy.

2.8 Applications of QET

Quantum Energy Teleportation has potential applications in quantum technologies, particularly in areas such as quantum computing and quantum communication. By allowing energy to be transferred between distant quantum systems without the physical transfer of particles, QET opens up possibilities for powering quantum devices remotely or facilitating energy-efficient quantum information processing. Moreover, studying QET in minimal systems, such as two-electron systems, helps to deepen our understanding of the interplay between entanglement, energy dynamics, and quantum mechanics.

3. Quantum Energy Teleportation in a Two-Electron System

In this section, we apply the Quantum Energy Teleportation (QET) protocol to a simplified system consisting of two entangled electrons. We aim to demonstrate how energy can be teleported between two spatially separated electrons via local operations and classical communication (LOCC). The calculations focus on the energy dynamics within the system, the conditions for successful energy extraction by Bob, and the effects of Alice's initial measurement.

3.1 Initial State and System Setup

We begin with two electrons in a maximally entangled **singlet state**:

$$\mid \psi \rangle = \frac{1}{2}(\mid 01\rangle - \mid 10\rangle),$$

where $|0\rangle$ *and* $|1\rangle$ denote the spin-down and spin-up states of the electrons, respectively. In this setup, Alice controls the electron at position n_A, and Bob controls the electron at position n_B. The system is assumed to be isolated from any external perturbations, and the total energy of the system is governed by the Hamiltonian H, which describes the interaction between the spins.

For the two-electron system, we assume a simple Hamiltonian of the form:

$$H = JS_A \cdot S_b,$$

where S_A *and* S_B are the spin operators for Alice's and Bob's electrons, respectively, and J is the coupling constant. The ground state of this system is the singlet state, and the energy associated with this state is $E_{ground} = -\frac{3}{4}J$

3.2 Alice's Local Measurement

The QET protocol begins with Alice performing a local projective measurement on her electron. For simplicity, we assume that Alice measures the spin of her electron along the z-axis, using the Pauli matrix ${\sigma_z}^A$. This measurement projects the electron into 1 of 2 possible states: $|1\rangle$ (spin-up) or $|0\rangle$ (spin-down).

Upon measurement, the entangled state collapses into a product state, depending on the outcome of Alice's measurement:

If Alice measures spin-up ($\mu = 1$), the post-measurement state will be::

$$|\psi_1{}'\rangle = |1\rangle_A \otimes |0\rangle_B.$$

If Alice measures spin-down ($\mu = 0$), the post-measurement state will be:

$$|\psi_0{}'\rangle = |0\rangle_A \otimes |1\rangle_B.$$

This measurement introduces energy into the system, as the ground-state entanglement is disturbed. The energy input by Alice, denoted E_A, is calculated as the difference in the expectation values of the Hamiltonian before and after the measurement:

$$E_A = \langle\psi'|H|\psi'\rangle - \langle\psi|H|\psi\rangle,$$

Since the post-measurement states $|\psi_1{}'\rangle$ and $|\psi_1{}'\rangle$ are product states with no entanglement, the energy of the system after Alice's measurement is zero. Thus, the energy input by Alice is:

$$E_A = 0 - (-\frac{3}{4}J) = \frac{3}{4}J$$

3.3 Classical Communication

After performing her measurement, Alice communicates the result μ (either 0 or 1) to Bob through a classical communication channel. This step ensures that Bob knows the outcome of Alice's measurement, allowing him to adjust his subsequent operation accordingly.

3.4 Bob's Local Unitary Operation

Once Bob receives the classical information from Alice, he performs a local unitary operation on his electron, tailored to the outcome of Alice's measurement. The goal of this operation is to extract energy from the system, making use of the entanglement between the electrons prior to Alice's measurement.

The local unitary operation performed by Bob is represented by:

$$V_B(\mu) = I\,cos\theta + i(-1)^{\mu}{\sigma_B}^x sin\theta,$$

where θ is a parameter determined by the correlation functions of the system, ${\sigma_B}^x$ is the Pauli matrix acting on Bobs electron, and μ is the result of Alice's measurement.

The parameter θ is chosen to maximize the energy extraction by Bob. It is related to the two-point correlation functions ξ and η, which describe the energy correlations between Alice's and Bob's electrons:

$$Cos(2\theta) = \frac{\xi}{\sqrt{\xi^2 + \eta^2}}, \quad Sin(2\theta) = -\frac{\eta}{\sqrt{\xi^2 + \eta^2}}$$

The correlation is defined as:

$$\xi = \langle \psi \mid \sigma^x{}_B H \sigma^x{}_B \mid \psi \rangle$$

$$\eta = \langle \psi \mid \sigma^z{}_A \sigma^{\cdot x}{}_B \mid \psi \rangle$$

where $\sigma^{\cdot x}{}_B$ represents the time derivative of Bobs spin operator in the Heisenberg picture, given by:

$\sigma^{\cdot x}{}_B = i[H, \sigma^x{}_B]$,

The operator $\sigma_B{}^x$ flips the spin of Bob's electron. The Hamiltonian in our system is:

$$H = JS_A \cdot S_B$$

In the singlet state $\mid \psi \rangle = \frac{1}{\sqrt{2}}(\mid 01 \rangle - \mid 10 \rangle)$, flipping Bob's spin would change the state to either $\mid 11 \rangle$ or $\mid 00 \rangle$, , both of which are perpendicular to the singlet state. Therefore, the expectation value of the Hamiltonian after Bob's spin flip is zero because the singlet state has no overlap with these orthogonal states. Hence, we have:

$$\xi = \langle \psi \mid \sigma^x{}_B H \sigma^x{}_B \mid \psi \rangle = 0$$

Next, η is calculated from the correlation between Alice's measurement and the time derivative of Bob's spin operator:

$$\eta = \langle \psi \mid \sigma^z{}_A \sigma^{\cdot x}{}_B \mid \psi \rangle$$

The time derivative of $\sigma_B{}^x$ is:

$$\sigma^{\cdot x}{}_B = i[H, \sigma^x{}_B]$$

Since the Hamiltonian involves the interaction between the spins $S_A \cdot S_B$ the commutator of H and $\sigma_B{}^x$ introduces a spin flip at Bob's site.

In the singlet state, Alice's spin and Bob's spin are perfectly anti-correlated. This means that when Alice's spin is measured along the z-Axis, Bob's spin is affected through the entanglement. The two-point function η captures this correlation, and since the singlet state is maximally entangled, the correlation is maximized at $\eta = -1$.

Thus, we have:

$$\eta = \langle \psi \mid \sigma^z{}_A \sigma^{\cdot x}{}_B \mid \psi \rangle = -1$$

3.5 Energy Extraction by Bob

Bob's energy gain, denoted E_B, is given by:

$$E_B = \frac{1}{2}(\sqrt{\xi^2 + \eta^2} - \xi)$$

Substituting the derived values $\xi = 0$ and $\eta = -1$ for the two-electron system, we find that:

$$E_B = \frac{1}{2}(\sqrt{0^2 + (-1)^2} - 0) = \frac{1}{2}J$$

Thus, Bob successfully extracts energy $E_B = \frac{1}{2}J$ from the system as a result of his local operation, enabled by the entanglement between the electrons.

3.6 Energy Conservation and Teleportation

The QET protocol ensures local energy conservation throughout the process. Initially, the system's total energy was $-\frac{3}{4}J$. After Alice's measurement, energy was introduced into the system, increasing the local energy around Alice's electron. Bob's operation results in the extraction of $E_B = \frac{1}{2}J$ from his local region, while the total energy of the system remains conserved due to the appearance of localized negative energy density near Bob's electron.

This negative energy density compensates for the energy extracted by Bob, maintaining the overall conservation of energy in the system. Thus, energy is effectively teleported from Alice's electron to Bob's electron through LOCC, without any physical transfer of particles.

III. RESULTS

Summary of Results

In this two-electron QET protocol, Alice inputs energy $E_A = -\frac{3}{4}J$ by performing a local measurement on her electron. Using the information communicated by Alice, Bob extracts $E_B = \frac{1}{2}J$ by performing a local unitary operation on his electron. The protocol respects local energy conservation, with energy being teleported from Alice's electron to Bob's electron via entanglement, classical communication, and local operations. The presence of negative energy density near Bob's electron ensures that the system's total energy remains conserved.

IV.References

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