ANALYSIS OF HIGH ORDER DISCRETE-TIME SYSTEMS USING IMPROVED ROUTH STABILITY METHOD via p-DOMAIN

M.Mallikaarjuna Reddy.* and M.Vjaya Raghavendra*
* Department of Electrical&Electronics Engg. , Sir C R Reddy College of Engineering.

Abstract : Reduction of High order system transfer function to lower order model has been an important subject area in the control engineering environment for many years. This paper proposes an approach for the analysis of high order discrete-time systems using modeling technique. The proposed order reduction method is **I**mproved Routh Stability method for the reduction of high order discrete-time systems via p-domain.
Keywords : Discrete System, Reduced Order Models, **I**mproved Routh Stability method.

Introduction : During last few decades a good deal of work has been reported in the literature to reduce the size of the high order model either in time domain and/or in frequency domain. The complexity of the system makes it difficult to obtain a good understanding of the original system and also the controller design for the system becomes a fairly computational tedious task. If the complexity of the system is minimized by reducing the order of the transfer function of the original system, then it is easier to understand the behavior of the original system. Many of the methods available for the analysis of systems are found to be more effective and computationally simple when applied for the systems of lower order. Where as application of these methods become computationally laborious when applied to practical systems, which are of high order. To overcome this problem the familiar procedure available in literature is that lower order models of high order systems are to be obtained and analyzed and those results are attributed for original high order systems. An approach for the analysis of high order discrete-time systems using modeling technique is proposed here. The proposed order reduction method is **I**mproved Routh Stability method for the reduction of high order discrete-time systems via p-domain.

Proposed procedure:

Consider the original high order discrete-time system defined as,

$$G(z)=\frac{b_0+b_1 z+b_2 z^2+\ldots\ldots}{a_0+a_1 z+a_2 z^2+\ldots\ldots\ldots\ldots}$$

Then the reduced model of order 'k' obtained by using proposed method is defined as,

$$R(z)=\frac{B_0+B_1 z+B_2 z^2+\ldots\ldots+B_{k-1} z^{k-1}}{A_0+A_1 z+\ldots\ldots+A_k z^k}$$

Applying linear transformation to G(z) i.e. substituting z= p + m, we find the result as,

$$G`(p)=\frac{D(p)}{E(p)}=\frac{d_{11}+d_{21}p+d_{12}p^2+\ldots\ldots}{e_{11}+e_{21}p+e_{12}p^2+\ldots\ldots}$$

where m = 1 for stable system with poles and zeros inside the unit circle.

'm' is a scalar quantity equal to distance from the farthest pole (zero) to the centre of unit circle for the systems having poles (zeros) lie outside the unit circle.

The α and β-parameters are obtained from the following arrays.

Denominator array:

e_{11}	e_{12}	e_{13}	e_{14}	……
e_{21}	e_{22}	e_{23}	e_{24}	……..
e_{31}	e_{32}	e_{33}	……	
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$e_{(n-1)1}$	$e_{(n-2)2}$	……		
e_{n1}	….			
$e_{(n+1)1}$	…..			

Numerator array:

d_{11}	d_{12}	d_{13}	d_{14}	….
d_{21}	d_{22}	d_{23}	d_{24}	….
d_{31}	d_{32}	d_{33}	…..	
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$d_{(m-1)1}$	$d_{(m-2)2}$	….		
d_{m1}	…	….		
$d_{(m+1)1}$	…..			

The elements other than first two rows of the above two tables are determined from the Routh-Hurwitz algorithm.

$$e_{i,j} = e_{(i-2),(j+1)} - \frac{e_{(i-2),1}\cdot e_{(i-1),(j+1)}}{e_{(i-1),1}}$$

$$d_{i,j} = d_{(i-2),(j+1)} - \frac{d_{(i-2),1}\cdot d_{(i-1),(j+1)}}{d_{(i-1),1}}$$

for i≥3 and i≤j≤(n-i-3)/2.

Then the desired α and β parameters are determined by the following relations.

$$\alpha_i = \frac{e_{i,1}}{e_{k+1,1}} \quad ; \quad i = 1,2,3,\ldots\ldots\ldots k+1$$

$$\beta_i = \frac{d_{i,1}}{d_{k,1}} \quad ; \quad i = 1,2,3,\ldots\ldots\ldots k$$

$$\gamma = \frac{G(z)}{\dfrac{\sum_{i=0}^{k-1} \beta_{i+1} \cdot (z-m)^i}{\sum_{i=0}^{k} \alpha_{i+1} \cdot (z-m)^i}} \quad ; \quad \text{for } z=1.$$

Alternatively, $\gamma = \frac{d_{k,1}}{e_{k+1,1}}$ for m=1, i.e. for stable system.

Now, the reduced model of order 'k' is

$$R_k(z) = \gamma \frac{B(z)}{A(z)}$$

where $B(z) = \sum_{i=0}^{k-1} \beta_{i+1} \cdot (z-m)^i$; and

$$A(z) = \sum_{i=0}^{k} \alpha_{i+1} \cdot (z-m)^i .$$

Example:

Consider original discrete time system represented by its transfer function,

$$G(z) = \frac{.3124\, z^3 - .5743\, z^2 + .3879\, z - .0889}{z^4 - 3.233\, z^3 + 3.9869\, z^2 - 2.2209\, z + .4723}$$

Then it is required to obtain the second order reduced model for the above system using the proposed order reduction method.

Applying linear transformation z=p+1;

$$G(p) = \frac{0.3124p^3 + 0.3629p^2 + 0.1765p + 0.0371}{z^4 + 0.767p^3 + 0.2879p^2 + 0.0539p + 0.0053}$$

The α,β parameters are:

$\alpha_1 = 0.0249$; $\alpha_2 = 0.254$; $\alpha_3 = 1$ and

$\beta_1 = 0.2102$; $\beta_2 = 1$

and γ at z=1 is $\gamma = 0.830667$

The second order reduced model using the proposed method is obtained as,

$$G_2(z) = \frac{0.830667\, z - 0.656062}{z^2 - 1.746329\, z + 0.771273}$$

The second order model derived by using Stability equation and weighted time moment method of R. Prasad is

$$G_2^1(z) = \frac{0.2766z - 0.107634}{z^2 - 1.775663\, z + 0.802801}$$

The unit step responses of original and reduced models are compared in Fig.1.

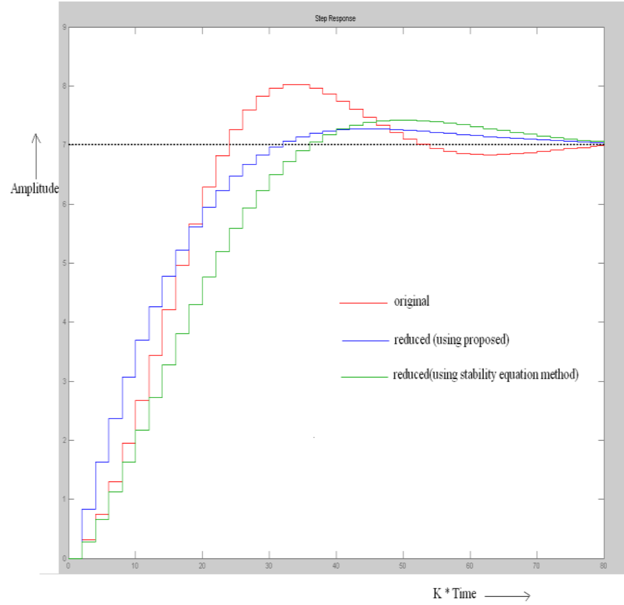

Fig.1

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