A Multi – Product EOQ Model Using Fuzzy For Three Parameter Weibull Distribution Deterioration with Budget Constraints and Demand Rate without Shortages

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Abstract - In this article, we examine a multi-product EOQ model for the deterioration of the three-parameter Weibull distribution with constant holding costs along with exponentially declining demand. When considering an EOQ model of inventory for goods with a linear rate of deterioration, the demand rate is taken into account as being reliant on the stock level at any given moment in time. This model strives to maximize the total cost. Shortages are not allowed when the demand rate increases gradually. Moreover, numerical examples are presented to illustrate the mathematical model. The result is illustrated with a graphical representation.

Index Terms - EOQ Model, Three parameter Weibull distribution, Demand rate, Fuzzy Model, Shortages.

I. INTRODUCTION

Weibull models are used to explain the phenomena and failures of the various sorts of observable working. They are frequently employed in analyses of profitability and dependability. A continuous probability distribution called the Weibull distribution is used to examine life data, model failure rates, and product reliability. Since it is a generalized gamma distribution with two shape parameters equal to k, it has two parameters, making it a two-parameter Weibull distribution. Because it can accurately predict the time to failure of actual, worldwide occurrences and is sufficiently adaptable despite having only two parameters, the Weibull distribution is particularly well-liked in survival analysis. The main goal of EOQ is to help businesses maintain consistent inventory levels and reduce costs. EOQ uses an annual usage cost that varies by order quantity and handling costs. Storage inventory can be costly for small business owners. The recent research work of this article in EOQ model is to minimize the total cost for weibull distribution deterioration item with inventory investment. The rate of deterioration may occur during the storage period of the units.

Covert R.P., and Philip G.C.,[7] Presented An EOQ Model for Items with weibull distribution deterioration. Further, Chakraborty, T., Giri, B.C. and Chaudhuri, K.S.[1] improved An EOQ model for items with Weibull distribution deterioration, shortage and trended demand. Moreover, Jalan, A.K., Giri, R.R. and Chaudhuri, K.S. [2] considered an EOQ model for items with Weibull distribution deterioration, shortage and ramp type demand. Later that Tripathy C.K and L.M. Pradhan [4] An EOQ model for three parameter Weibull deterioration with permissible delay in payments and associated salvage value. Amutha.R and Chandrasekaran.E [6] “Deteriorating Inventory Model for Two Parameter Weibull Demand with Shortages.

In general, numerous researchers have worked in the area of quadratic demand, price dependent demand, Weibull distribution deterioration and power backlogged. Numerical example and graphical diagram were used to verify the problem.

II. ASSUMPTIONS AND NOTATIONS

The inventory model is developed based on the following assumptions and notations.

ASSUMPTIONS

- **(i)** All products having an equal replenishment cycle of length T. This practice is often followed in multi-product inventory models. If a multi-product venture initiates procurement actions for different products at different times, the situation will be fixed in terms of operations and installation costs will be incurred each time. When a purchase action is performed.
- **(ii)** Replenishment rate is instantaneous with zero time.

(iii) Shortages are allowed.

(iv) The demand rates are increases exponentially and for i^{th} item is given by the function $R_i=ae^{bt}$, a>0,0<b<1.

(v) The rate of deterioration at any time t>0 follow the three parameter weibull distribution as Θi = 〚αβ(t-γ)〛^(β-1) where α (0<α<1) is the scale parameter, β(>0) is the shape parameter and γ<0 is the location parameter.

NOTATIONS

n = total number of products produced by the company

R_i = Demand rate

$q_{i=}$ Order Size of the product i

$Q_i(t)$ = The inventory of the item i, at any time t

C_{11}= Holding cost per unit time

C_{12} = Shortage cost per unit time

C_{13} =Setup Cost per production run

f_i = The Shortage space of the item i, per unit time

P_i= Unit selling price

F = Total Fixed cost

A = Total storage space

M = Capital amount invested

Θ_i= Fraction of inventory per unit time

C(q,s,t) = The total Cost of the product

III. DEVELOPMENT OF THE MODEL

The stock quantity of the i^{th} item is q_i at time t = 0. At intervals of [0, T], inventory gradually decreases, mainly to meet demand and partly due to deterioration. This process allows inventory to go to zero at time t_1 (bottlenecks to occur at intervals of [1, T]. Then this cycle was repeated. The shortage of the i^{th} item is S_i (i = 1, ... n) at time t = T. The differential equation of the instantaneous stock Q_i (t) of the i^{th} item at any time t of [0, T] with respect to the exponential demand and the deterioration of the Weibull distribution are given by the following equation**.**

$$\frac{dQ_i(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1} Q_i(t) = -ae^{bt}, t_1 \leq t \leq 0 \tag{1}$$

$$\frac{dQ_i(t)}{dt} = -ae^{bt}, T \leq t \leq t_1 \tag{2}$$

Boundary conditions are

$$Q_i(0) = q_i\,;\;\; Q_i(t_i) = 0 \;\text{ and }\; Q_i(T) = -S_i \tag{3}$$

∴The Differential equations are

$$Q_i(t) = \left\{q_i - a\left(t + b/2t^2 + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha b(t_1-\gamma)^{\beta+2}}{\beta+2}\right)\right\} e^{-a(t-\gamma)^{\beta}}, 0 \leq t \leq t_1 \tag{4}$$

$$\text{And } Qi(t) = \frac{a}{b}\{e^{bt}{}_1 - e^{bt}\}, t \leq t_1 \leq T \tag{5}$$

Since t=t_1 by using boundary conditions we can get

$$q_i = a\left(t_1 + b/2\, t_1{}^2 + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha b(t_1-\gamma)^{\beta+2}}{\beta+2}\right) \tag{6}$$

Total Inventory of i^{th}item in $[0,t_1]$ is

$H_i = \int_0^{t_1} Q_i(t)dt$

$$H_i = q_i\left(t_1 - \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right) + a\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t-\gamma)^{\beta+3}}{\alpha(\beta+2)(\beta+3)}\right) \quad (7)$$

Total Deterioration of i^{th}item in $[0,t_1]$ is

$F_i = \int_0^{t_1} \theta_i\, Q_i(t)dt$

$$F_i = q_i\,\alpha(t_1-\gamma)^{\beta} + a\left(\frac{\alpha\beta(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha\beta b(t_1-\gamma)^{\beta+2}}{\beta+2}\right) \quad (8)$$

Thus, Total shortage of the i^{th}item in $[0,t_1]$ is

$D_i = \int_{t_1}^{T}\{-Q_i(t)dt\}$

$$D_i = \frac{S_1^{\,2}e^{bt_1}}{2a} \quad (9)$$

The total cost of n- items is,

C(q,s,T) = (Setup cost + Production cost + Inventory holding cost + Deterioration cost + shortage cost) for n-items + Fixed cost

$$C(q, s, T) = \sum_{i=1}^{n}\left[c_{3i} + \left\{r_i + \alpha r_i(t_1-\gamma)^{\beta} + c_{1i}\left(t_1 - \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right)\right\}q_i + ac_{1i}\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{\alpha(\beta+2)(\beta+3}\right) + a\alpha\beta\left(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\right)r_i + \frac{c_{2i}S_i^{\,2}e^{bt_1}}{2a}\right] + FT$$

Total cost per unit cycle is

$$C(q, s, T) = \frac{1}{T}\sum_{i=1}^{n}\left[c_{3i} + \left\{r_i + \alpha r_i(t_1-\gamma)^{\beta} + c_{1i}\left(t_1 - \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right)\right\}q_i + ac_{1i}\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t_1-\gamma)^{\beta+2}}{\alpha(\beta+2)(\beta+3)}\right) + a\alpha\beta\left(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\right)r_i + \frac{c_{2i}S_i^{\,2}e^{bt_1}}{2a}\right] + F \quad (10)$$

Now our object is to minimize the total cost subject to the storage and inventory investment constraints. Then the problem is as

$$\text{Minimize } c(q, s, T) = \frac{1}{T}\sum_{i=1}^{n}\left[c_{3i} + \left\{r_i + \alpha r_i(t_1-\gamma)^{\beta} + c_{1i}\left(t_1 - \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right)\right\}q_i + ac_{1i}\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3)}\right) + a\alpha\beta\left(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\right)r_i + \frac{c_{2i}S_i^{\,2}e^{bt_1}}{2a}\right] + F \quad (11)$$

subject to

$$\sum_{i=1}^{n} f_i q_i \le A \quad (12)$$

and $\sum_{i=1}^{n} r_i q_i \le M$ (13)

Where $q_i \ge 0, s_i \ge 0, T \ge 0$

Again our object is to maximize the total cost subject to the shortage and inventory investment constraints.

The problem is as Maximize

$$c(q, s, T) = \frac{1}{T}\sum_{i=1}^{n}\left[c_{3i} + \left\{r_i + \alpha r_i(t_1-\gamma)^{\beta} + c_{1i}\left(t_1 - \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\right)\right\}q_i + ac_{1i}\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3)}\right) + a\alpha\beta\left(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\right)r_i + \frac{c_{2i}S_i^{\,2}e^{bt_1}}{2a}\right] + F \quad (14)$$

subject to

$\sum_{i=1}^{n} f_i q_i \le A$ and (15)

$$\sum_{i=1}^{n} r_i q_i \le M \quad (16)$$

IV. FUZZY MODEL

Let the demand and deterioration are as$R_i = ae^{bt}$ a>0, 0<Θ<1 and

$\theta_i = \alpha\beta(t-\gamma)^{\beta-1}$, 0<α<1, β>0, we consider 'a' in demand as a triangular fuzzy number,

We consider "a" in demand as a triangular fuzzy number and defined as $a^{\sim} = (a_1, a_2, a_3)$. The corresponding membership function is defined as

$$\mu_{a^{\sim}}(x) = \frac{x-a_1}{a_2-a_1}$$
$$= \frac{a_3-x}{a_3-a_2}$$

α – Cut of a fuzzy set A(x)

This is the set consisting of elements x of the universal set X, whose membership values are neither greater nor equal to the value of α . It's denoted by the symbol ${}^{\alpha}\mu_A(x)$ and is defined as

${}^{\alpha}\mu_A(x) = \{x/\mu_A(x) \geq \alpha\}$. Assume α cut as $a^{\sim}$, Let us consider that η - cut as α-cut and we have

$\mu_{a^{\sim}(x)} \geq \eta, 0 \leq \eta \leq 1.$

Then, $\frac{x-a_1}{a_2-a_1} \geq \eta$and $\frac{a_3-x}{a_3-a_2} \geq \eta$(i.e) $x \geq a_1 + \eta(a_2-a_1)$ and $x \leq a_3 + \eta(a_3-a_2)$.

So an η – cut of can be $a^{\sim}$ expressed by the following interval$a^{\sim}(\eta) = [a_1 + \eta(a_2-a_1), a_3 - \eta(a_3-a_2)]$,$\eta\epsilon[0,1]$ $where a^{-}(\eta) = a_1 + \eta(a_2-a_1)$

And $a^{+}(\eta) = a_3 + \eta(a_3-a_2) \Rightarrow upper\ cut$ are called as lower and upper cut respectively.

The system of equation for the given problem in [1,0] as

$$\frac{dQ_i(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1}Q_{i(t)} = -a^{\sim}(\eta)e^{bt}, 0 \leq t \leq t_1 \quad (17)$$

$$\frac{dQ_i(t)}{dt} = a^{\sim}(\eta)e^{bt}, t_1 \leq t \leq T \quad (18)$$

$$\frac{dQ_i{}^{+}(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1}Q_i(t) = -a^{-}(\eta)e^{bt} \quad (19)$$

$$\frac{dQ_i{}^{-}(t)}{dt} + \alpha\beta(t-\gamma)^{\beta-1}Q_i(t) = -a(\eta)e^{bt} \quad (20)$$

And

$$\frac{dQ_i(t)^{-}}{dt} = -a(\eta)e^{bt}, t_1 \leq t_i \leq T \quad (21)$$

$$\frac{dQ_i{}^{-}(t)}{dt} = +a(\eta)e^{bt} \quad (22)$$

Where $a^{-}(\eta) = a_1 + \eta(a_2-a_1)$ and $a^{+}(\eta) = a_3 - \eta(a_3-a_2)$ are as lower and upper cut.

The differential equations of solution (16) & (17) are given by

$$Q_i{}^{+}(t) = \left\{q_i + a^{-}(\eta)\left(t + b/2\,t^2 + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha b(t_1-\gamma)^{\beta+2}}{\beta+2}\right)\right\}e^{a(t-\gamma)^{\beta}}, \quad 0 \leq t \leq t_1 \quad (23)$$

$$Q_i{}^{-}(t) = \left\{q_i + a^{+}(\eta)\left(t + b/2\,t^2 + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha b(t_1-\gamma)^{\beta+2}}{\beta+2}\right)\right\}e^{-a(t-\gamma)^{\beta}}, 0 \leq t \leq t_1 \quad (24)$$

And also,

$$Q_i^{+}(t) = \frac{a^{-}(\eta)}{b}\{e^{bt_1} - e^{bt}\}, t_1 \le t \le T \tag{25}$$

$$Q_i^{-}(t) = \frac{a^{+}(\eta)}{b}\{e^{bt_1} - e^{bt}\}, t_1 \le t \le T \tag{26}$$

At t=t_1 by using boundary condition and then from (20) & (21) we can get

$$q_i = -a^{-}(\eta)\left(t_1 + b/2\, t_1^{2} + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha b(t_1-\gamma)^{\beta+2}}{\beta+2}\right) \tag{27}$$

$$q_i = -a^{+}(\eta)\left(t_1 + b/2\, t_1^{2} + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha b(t_1-\gamma)^{\beta+2}}{\beta+2}\right) \tag{28}$$

Now, The total upper inventory of i^{th} item in $[0,t_1]$ is

$$H_i^{+} = \int_0^{t_1} Q_i^{+}(t)dt$$

$$H_i^{+} = q_i\left(t_1 - \frac{\propto(t_1-\gamma)^{\beta+1}}{\beta+1} + a^{-}(\eta)\left(\frac{t_1^{2}}{2} + \frac{bt_1^{3}}{b} - \frac{\alpha\beta(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t-\gamma)^{\beta+3}}{\alpha(\beta+2)(\beta+3)}\right)\right) \tag{29}$$

Then, the total lower inventory of i^{th}item in $[0,t_1]$ is

$$H_i^{-} = \int_0^{t_1} Q_i^{-}(t)dt$$

$$H_i^{-} = q_i\left(t_1 - \frac{\propto(t_1-\gamma)^{\beta+1}}{\beta+1} + a^{+}(\eta)\left(\frac{t_1^{2}}{2} + \frac{bt_1^{3}}{b} - \frac{\alpha\beta(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b(t-\gamma)^{\beta+2}}{\alpha(\beta+2)(\beta+3)}\right)\right) \tag{30}$$

Thus, Total upper deterioration of i^{th} item in $[0,t_1]$ is

$$F_i^{+} = \int_0^{t_1} \theta_i Q_i^{+}(t)dt$$

$$F_i^{+} = q_i\alpha(t_1-\gamma)^{\beta} + a^{-}(\eta)\left(\frac{\alpha\beta(t_1-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha\beta b(t_1-\gamma)^{\beta+2}}{\beta+2}\right) \tag{31}$$

Total lower deterioration of i^{th} item in $[0,t_1]$ is

$$F_i^{-} = \int_0^{t_1} \theta_i Q_i^{-}(t)dt$$

$$F_i^{-} = q_i\alpha(t_1-\gamma)^{\beta} + a^{+}(\eta)\left(\frac{\alpha\beta(t_1-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha\beta b(t_1-\gamma)^{\beta+2}}{\beta+2}\right) \tag{32}$$

Total upper shortage of i^{th} item in $[0,t_1]$ is

$$D_i^{+} = \int_{t_1}^{T}\{Q_i^{+}(t)\}dt$$

$$D_i^{+} = \frac{S_i^{2}e^{+bt_1}}{2a^{-}(\eta)} \tag{33}$$

Total upper shortage of i^{th} item in $[0,t_1]$ is

$$D_i^{+} = \int_{t_1}^{T}\{-Q_i^{-}(t)\}dt$$

$$D_i^{-}=\frac{S_i^{2}e^{+bt}}{2a^{+}(\eta)} \quad (34)$$

Total upper cost of n- items is C(q,s,T)= (Setup Cost+ Production cost + Upper inventory holding cost+ Upper deterioration cost+Upper shortage cost) for n- items+ Fixed cost

$$C(q,\ s,\ T) = \sum_{i=1}^{n}\Big[c_{3i}+\Big\{r_i+\alpha r_i(t_1-\gamma)^{\beta}+c_{1i}\Big(t_1-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\Big)\Big\}q_i+\ a^{-}(\eta)c_{1i}\Big(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3}\Big)+a^{-}(\eta)\alpha\beta\Big(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\Big)r_i+\frac{c_{2i}S_i^{2}e^{bt_1}}{2a}\Big]+FT$$

Where $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\ \eta(a_3-a_2)$.

Total lower cost of n- items is C(q,s,T)= (Setup Cost+ Production cost +lower inventory holding cost+ lower deterioration cost + lower shortage cost) for n- items+ Fixed cost

$$C(q,\ s,\ T) = \sum_{i=1}^{n}\Big[c_{3i}+\Big\{r_i+\alpha r_i(t_1-\gamma)^{\beta}+c_{1i}\Big(t_1-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\Big)\Big\}q_i+a^{+}(\eta)c_{1i}\Big(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3}\Big)+a^{+}(\eta)\alpha\beta\Big(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\Big)r_i+\frac{c_{2i}S_i^{2}e^{bt_1}}{2a}\Big]+FT$$

The problem is as minimize the total upper cost

$$C(q,\ s,\ T) = \sum_{i=1}^{n}\Big[c_{3i}+\Big\{r_i+\alpha r_i(t_1-\gamma)^{\beta}+c_{1i}\Big(t_1-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\Big)\Big\}q_i+a^{-}(\eta)c_{1i}\Big(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3}\Big)+a^{-}(\mu)\alpha\beta\Big(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\Big)r_i+\frac{c_{2i}S_i^{2}e^{bt_1}}{2a}\Big]+F \quad (35)$$

subject to

$$\sum_{i=1}^{n} f_i q_i\ \le\ A \quad (36)$$

$$\sum_{i=1}^{n} r_i q_i\ \le\ M \quad (37)$$

To minimize the lower cost the problem is as

$$Min(q,s,T)= \frac{1}{T}\sum_{i=1}^{n}\Big[c_{3i}+\Big\{r_i+\alpha r_i t_1^{\beta}+c_{1i}\Big(t_1-\frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}\Big)\Big\}q_i+a^{+}(\eta)c_{1i}\Big(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b(t_1-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3}\Big)+a^{+}(\mu)\alpha\beta\Big(\frac{(t_1-\gamma)^{\beta+1}}{\beta+1}+\frac{b(t_1-\gamma)^{\beta+2}}{2(\beta+2)}\Big)r_i+\frac{c_{2i}S_i^{2}e^{bt_1}}{2a}\Big]+F \quad (38)$$

Subject to

$$\sum_{i=1}^{n} f_i q_i\ \le\ A \quad (39)$$

$$\sum_{i=1}^{n} r_i q_i\ \le\ M \quad (40)$$

Where $q_i\ge 0, S_i\ge 0, T\ge 0\ and\ a^{+}(\eta)=a_3-\ \eta(a_3-a_2)$

V. NUMERICAL EXAMPLES

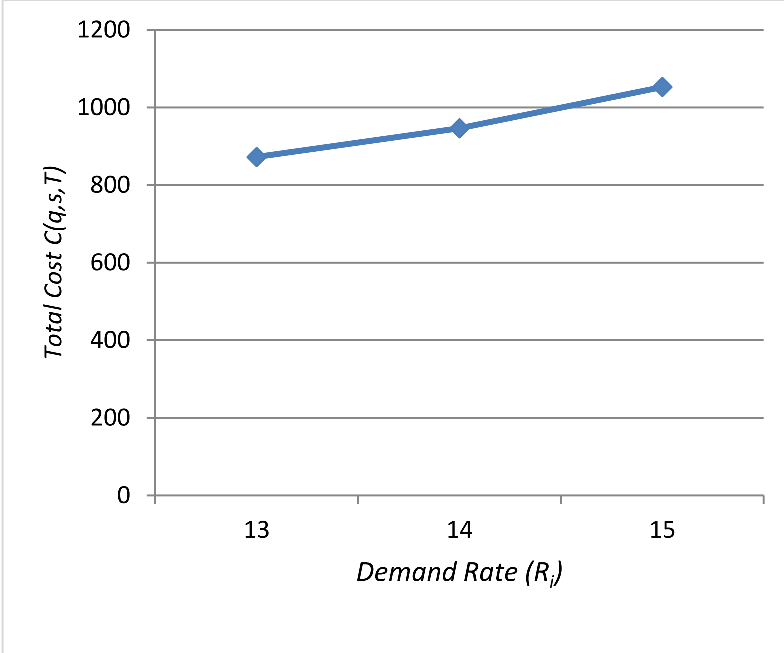
It is considered that the values corresponding to the above-mentioned problem variables are as follows

[1].Let us consider that f1=\$20, f2= \$22, α=0.06, β=1.5, γ= -0.5,a=100, b=0.03, then the total storage space is A=\$1400, M=\$1000 of the Capital amount invested . Then C11=\$5, C12=\$4.5, C21= \$12, C22=\$14.7, C31= \$100, C32=\$100, T=1, r1=\$12,r2=\$14,S1=3, S2=6 and the fixed cost F=\$50. Hence the total cost of the product C(q,s,T) = \$872.514.

[2].Let us consider that f1=\$18, f2= \$20, α=0.06, β=1.8, γ= -0.8,a=100, b=0.03, then the total storage space is A=\$1100, M=\$1000 of the Capital amount invested . Then C11 =\$5,C12=\$4, C21= \$12.5,C22=\$13, C31= \$100, C32=\$100, T=1, r1=\$13,r2=\$14,S1=4, S2=\$8 and the fixed cost F=\$50. Hence the total cost of the product C(q,s,T) = \$946.271.

[3].Let us consider that f1=\$22, f2= \$24, α=0.06, β=1.7, γ= -1,a=100, b=0.03, then the total storage space is A=\$1800, M=\$1000 of the Capital amount invested . Then C11 =\$5,C12=\$3.5, C21= \$13,C22=\$14, C31= \$100, C32=\$100, T=1, r1=\$14,r2=\$16,S1=6, S2=\$9 and the fixed cost F=\$50. Hence the total cost of the product C(q,s,T) = \$1052.59.

Relationship between Demand Rate and Total cost



VI. CONCLUSION

In this paper, we examine a multi-product EOQ model using Fuzzy for an exponentially increasing demand and three-parameter Weibull distribution deterioration with constant holding cost. This model concludes that the total cost of the product increases when the demand rate increases gradually. The aforementioned numerical example proved and successfully shown. This article's contribution is the creation of a mathematical model and a successful approach to finding the ideal solution.

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