Indian Contribution in Mathematical Development

Dr. Pushpander Kadian, Dr. Jaibhagwan, Dr. Parvesh Kumar
[1] Associate Professor, Govt. PG Nehru College, Jhajjar, India
[2] Associate Professor, Pt. NRS Govt. College, Rohtak, India
[3] Associate Professor, Pt. NRS Govt. College, Rohtak, India.

ABSTRACT

Mathematics is not only the study of numbers, counting and measuring, but it involves the study of number patterns and relationships, also. It is a way to communicate thoughts and is a way of reasoning that is distinctive to human beings. Ancient Indian mathematicians made very significant contributions to the discipline of mathematics. In India, Mathematics has its roots in Vedic literature authored by Indian mathematicians thousands of years ago where we see for the first time, the concept of zero, the techniques of Algebra and algorithm, square root and cube root. Indians in ancient times also made advances in abstract sciences like Mathematics and Astronomy. It has now been generally accepted that the technique of Algebra and the concept of zero is attributed to Indians. This contribution exceeds all others as it forms the foundation of the decimal number system, without which no mathematical growth would have been conceivable. The modern number system was established by Indians, and it is still known as Indo-Arabic numerals. Even the technique of calculation, called algorithm, which is today widely used in designing software programs for computers was also derived from Indian Mathematics**.** India has been the home to some of the world's most brilliant brains. India has been a pioneer in the field of Mathematics and science from the ancient time. In this paper, we have presented a brief account of Indian contribution in the field of mathematics starting from Indus Valley Civilization (3000 BC) upto 20th century. The journey of Indian mathematicians from ancient time to the Modern Era, passing through the Classical Era of Indian Mathematicians (500 to 1200 AD), has been presented in detail.

Keywords: Indo-Arabic numerals, Classical Era, Shulba Sutras, Brahmi Numerals, The place-value system etc.

INTRODUCTION

Mathematics plays a vital role in the modernization of society as it is everywhere and affects the everyday lives of people. Although it is abstract and theoretical knowledge, it emerges from the real world. It is also a way to correspond and evaluate ideas, a tool for organizing and interpreting data and above all, a method of logical reasoning unique to man. Mathematics has been existed since the early age of human civilization. The historians who exploited Indian history literature at ancient time were not always perfect in their methods of investigation and consequently promulgated many errors. The early chronology has been largely revised and the revision in some instances has important bearings on the history of mathematics and allied subjects. According to orthodox Hindu tradition the Surya Siddhanta, the most important Indian astronomical work, was composed over two million years ago! Bailly, towards the end of the eighteenth century, considered that Indian astronomy had been founded on accurate observations made thousands of years before the Christian era. Laplace, basing his arguments on figures given by Bailly, considered that some 3,000 years B.C., the Indian astronomers had recorded actual observations of the planets correct to one second. Later on, with the researches of Colebrooke, Whitney, Weber, Thibaut, and several others, more correct views were introduced and it was proved that the records used by Bailly were quite modem and that the actual period of the composition of the original Surya Siddhanta was not earlier than 400 A.D. It may, indeed, be generally stated that the tendency of the early historians was towards antedating and this tendency is exhibited in discussions connected with two notable works, the Sulvasutras and the Bakhshali arithmetic, the dates of which are not even yet definitely fixed. In this paper we have studied the contribution of Indian mathematicians starting from the Indus Valley Civilisation to the modern era of mathematics. The metallic seals and archeological remains of earliest civilization establishes the mathematical knowledge of Indian mathematicians at that time. The Brahmi Numerals, the place-value system and the concept of Zero, given by Indian mathematicians, has been depicted in this paper. The marvelous work by Aryabhata I and Varahamihira (500 AD), Bhaskara I (600 AD), Brahmagupta (700 AD), Mahavira (800 AD), Aryabhatta II (1000 AD) and Bhaskarachrya or Bhaskara II (1200 AD) is also being presented in detail. The work done by the modern era mathematicians like Kaprekar (1905-1988), Harish-Chandra (1923-1983), Shakuntla Devi (1929-2013) and one of the greatest modern era mathematicians Srinivasa Ramanujan (1887-1920) is also being presented.

THE JOURNEY OF MATHEMATICS IN INDIA

Mathematics has played a very significant role in the progress and expansion of Indian culture for centuries. Mathematical ideas that originated in the Indian subcontinent have had a thoughtful impact on the world. In ancient time, mathematics was primarily used in a supplementary or practical role. Thus mathematical methods were used to solve problems in architecture and construction (which is evident in the public works of the Harappan civilization), in astronomy, astrology and in the construction of Vedic altars (as in the case of the Shulba Sutras of Baudhayana and his successors). By the sixth or fifth century BC, mathematics was studied for its own sake, as well as for its relevance in other fields of knowledge. In fact there was not the single period in Indian history when mathematics was not being developed and included in the lives of the people. Here, we present the development of mathematical concepts in India starting from the ancient time to the modern day.

1. Mathematics in ancient times (3000 to 600 BC)

Indus Valley Civilization is the earliest and the oldest confirmation of Indian mathematical understanding and its application. The metallic seals found in the excavations of Mohenjo-Daro and Harappan indicates that the people of this civilization had the knowledge of numbers. It is also understandable from the pottery and other archaeological remains that they had the acquaintance of dimensions and geometry even in crude form. The Indus valley civilization existed around 3000 BC. Two of its most famous cities, Harappa and Mohenjo-Daro, present authentication that the construction of buildings followed a standardized measurement which was decimal in nature.

Vedic Mathematics is the most importance era in the development of the Indian mathematical ideas. In particular, the Shatapatha Brahmana, which is a part of the Shukla Yajur Veda, includes comprehensive descriptions of the geometric construction of altars for yajnas. In this period, the brick making technology of Indus Valley civilization extended to new uses. Among the scholars of the post-Vedic period who contributed to mathematics, the most notable is Pingala (300–200 BC), who authored the Chhandas Shastra, a sanskrit treatise on prosody. There is evidence that in his work on the enumeration of syllabic combinations, Pingala stumbled upon both Pascal Triangle and binomial coefficient. Pingala's work also contains the basic ideas of Fibonacci numbers.

Shulba Sutras are complementary to the Vedas. These texts are considered to date from 800 to 200 BC. Four in numbers, they are named after their authors: Baudhayana (600 BC), Manava (750 BC), Apastamba (600 BC), and Katyayana (200 BC). The sutras hold the famous theorem usually attributed to Pythagoras. The Shulba Sutras initiated the concept of irrational numbers. Mathematical development of this period was associated with the solution of practical geometric problems, particularly the construction of religious altars. But, one can found a hint of the development of the series expansion which illustrates towards the development of an algebraic point of view. In later times, we locate a move towards algebra, with simplification of algebraic formulae and summation of series acting as catalysts for further mathematical discovery.

2. Mathematics during Medieval period (600 BC to 500 AD)

Just as Vedic philosophy and theology encouraged the development of positive aspects of mathematics, so did the rise of Jainism. Jain cosmology showed the way to ideas of the infinite. This in turn, led to the development of the notion of orders of infinity as a mathematical concept. By orders of infinity, we indicate a theory by which one set could be deemed to be 'more infinite' than another. In modern language, this matches to the concept of cardinality. In Europe, it was not until Cantor's effort in the nineteenth century that an appropriate concept of cardinality was recognized. Besides the investigations into infinity, this period saw developments in several other fields such as number theory, geometry, computing, with fractions etc. In particular, the recursion formula for binomial coefficients and the 'Pascal's triangle' were already known in this period.

The period 500 AD corresponds with the beginning and supremacy of Buddhism. In the Lalitavistara, a memoir of the Buddha which may have been written around the first century AD, there is an incident about Gautama was asked to name of large powers of 10 starting with 10. He is able to give names to numbers up to 10 (tallaksana). This incident clearly depicts that the mathematicians of that period were capable of telling very large and big numbers. And it is also a reality that these large numbers cannot be calculated without any proper or at least some type of place value system.

Indian Mathematical development and contribution will always be incomplete without discussing the Indian numerals and the Place-Value system. The numbers that are in practice today can be marked out to the Brahmi numerals that appear to have made their emergence in 300 BC. But Brahmi numerals were not part of a place value system. They developed into the Gupta numerals around 400 AD and afterward into the Devnagari numerals, which developed gradually between 600 and 1000 AD.

The place-value decimal system was first introduced in India. It would be adequate to mention here an often-quoted comment by Laplace (1749-1827), "It is India that gave us the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a thoughtful and important idea which appears so simple to us now that we ignore its true merit. But it's very simplicity and the great ease which it has lent to computations put our arithmetic in the first rank of useful inventions; and we shall appreciate the magnificence of the achievement the more when we remember that it escaped the genius of Archimedes and Apollonius, two of the greatest men produced by ancient times". At the same time, other civilizations were also using the place-value system based on numbers; for example, the Babylonians used a sexagesimal place-value system as early as 1700 BC, but the Indian system was the first decimal system. Moreover, until 400 BC, The Babylonian system had an innate vagueness as there was no symbol for zero. Thus it was not a complete place-value system in the manner we assume of it today.

3. The Classical Era of Indian Mathematics (500 AD to 1200 AD)

There was time in the Indian mathematical development which can be called the classical era of Indian Mathematics as the most famous and significant names of Indian mathematicians are from this period and these mathematicians established India as the source of science and mathematics. This can be seen in the words of Albert Einstein, German scientist and humanist (1879-1955), "We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made." The great Indian mathematicians Aryabhata I and Varahamihira (500 AD), Brahmagupta (700 AD), Bhaskara I (900 AD), Mahavira (900 AD), Aryabhatta II (1000 AD) and Bhaskarachrya or Bhaskara II (1200 AD) all belonged to this golden age.

Kusumapura near Pataliputra and Ujjain emerged as the two centers of mathematical research at this time. Aryabhata I was the leading figure at Kusumapura. The most significant discovery to which the whole world is indebted is that of numeral Zero. After the introduction of zero by Indians, the rise of zero as the equal position as other numbers (numerals), created problems for the several bright mathematicians and they all struggled with the concept of zero initially. The key dilemma included the formulation of such arithmetic system which include zero. While addition, subtraction, and multiplication with zero were mastered, division was a more restrained question. Today, we know that division by zero is not well-defined and so has to be excluded from the rules of arithmetic. But this perception was not or cannot be achieved at once at that time as it was totally a new idea for the whole world. This problem took the collective efforts of many minds. It is fascinating to note that it was not until the seventeenth century that zero was being used in Europe.

Aryabhata also gave the technique for solving linear equations of the form ax + by = c. He devised a general method for solving such type of equations, and he called it the kuttaka method. It must be understood that Aryabhata calculated linear equations because of his interest in astronomy. His other contribution includes approximation of Pie to four decimal places (3.1416) and work on trigonometry. At the time, he wrote Aryabhattiya, a mathematical compendium which is divided into four sections. His work explains how to represent huge decimal numbers using alphabets. It also contains difficult questions from contemporary fields of mathematics such as number theory, geometry, trigonometry, and algebra. He correctly asserted, contrary to popular opinion at the time, that the planet rotates on its axis on a regular basis and that the apparent movement of the stars is produced by the rotation of the globe.

Another most important centre of mathematical learning during this phase was Ujjain, which was home to Varahamihira, Brahmagupta and Bhaskaracharya. One of Varahamihira's most notable works was the Brihat Samhita, an encyclopaedic study of architecture, temples, planetary motions, eclipses, timekeeping, astrology, seasons, cloud formation, rainfall, agriculture, arithmetic, gemology, scents, etc. His other notable works include the discovery of trigonometric equations like $sin^2x + cos^2x = 1 \; and \; cos\left(\frac{\pi}{2} - x\right) = \sin x \;\; etc$ and improving the precision of Aryabhata's sine tables, the algebraic properties of zero and negative numbers.

Bhaskara I was an Indian mathematician and astronomer who was the first to write numbers in the Hindu-Arabic decimal system with a circle for the zero, and who gave a unique and remarkable rational approximation of the sine function in his commentary on Aryabhata's work. This commentary, Aryabhatiyabhasya, written in 629 AD, is among the oldest known prose works in sanskrit on mathematics and astronomy. He also wrote two astronomical works in the line of Aryabhata's school: the Mahabhaskariya (Great Book of Bhaskara) and the Laghubhaskriya (Small Book of Bhaskara).

The text Brahmasphutasiddhanta by Brahmagupta, published in 628 AD, dealt with arithmetic involving zero and negative numbers. Like Aryabhata, Brahmagupta was an astronomer, and he was greatly influence by the astronomy and this interest encouraged him to work in the field of mathematics. He solved the difficulties of astronomy using the mathematical concepts. He presented the well-known formula for a solution to the quadratic indeterminate equations. He also studied quadratic equation $ax^2 \pm c = y^2$ in two variables and sought solutions in whole numbers. Brahmagupta put forward a method, which he named samasa, by which; known two solutions of the equation a third solution could be created. Brahmagupta's lemma was acknowledged one thousand years before it was rediscovered in Europe by Fermat, Legendre, and others.

During this period, Mathematics not only proliferated in North and middle but in South India also. Mahavira (900 AD) was a mathematician who belongs to the ninth century who has been most likely from modern day Karnataka. He calculated the problem of cubic and quartic equations and solved them for some families of equations. Mahavira asserted that the square root of a negative number did not exist. He gave the sum of a series whose terms are squares of an arithmetical progression, and gave empirical rules for area and perimeter of an ellipse. His work had a considerable impact on the development of mathematics in South India. His book Ganita Sara-Sangraha on numerical mathematics, intensified the discoveries and the researches of Brahmagulpta and proposed a very constructive orientation for the position of mathematics at that time.

Aryabhata's II (1000AD) most eminent work was Maha Siddhanta. The treatise consists of eighteen chapters and was written in the form of verse in Sanskrit. The initial twelve chapters deals with topics related to mathematical astronomy and covers the topics that Indian mathematicians of that period had already worked on. The various topics that have been included in these twelve chapters are the longitudes of the planets, lunar and solar eclipses, the estimation of eclipses, the lunar crescent, the rising and setting of the planets, association of the planets with each other and with the stars. The next six chapters of the book include topics such as geometry, geography and algebra, which were applied to calculate the longitudes of the planets. In about twenty verses in the treatise, he gives elaborate rules to solve the indeterminate equation $by = ax + c$. These rules have been applied to a number of different cases such as when c has a positive or negative value or when the number of the quotients is an even number or odd number etc.

The classical era closes with Bhaskaracharya or Bhaskara II (1100 AD). Siddhanta Siromani, his main work, is divided into four parts, which are frequently regarded as four different works and are titled as Lilavati (Arithmetic and measurement), Bijaganita (Algebra), Ganitadhyaya and Goladhyaya (Astronomy). In that sequence, these four parts address arithmetic, algebra, planetary mathematics, and spheres. In his original work on arithmetic (Lilavati), he advanced the kuttaka method of Aryabhata and Brahmagupta. The Lilavati is remarkable for its originality and diversity of topics. It also explains how to solve indeterminate quartic, cubic, and quadratic equations. Bhaskara developed the first calculus about 500 years before Newton and Leibniz and Calculated derivatives for trigonometric formulae and functions.

4. Mathematics in the Modern Era (1200 AD onwards)

Indian mathematical development does not end with the classical era. In fact it moves ahead with the mathematicians of modern age who were and are equally competent. Most of the work during this period is attributed to the mathematicians of South India. Madhava of Sangamagrama (1340-1425) from Kerala was remarkable mathematician of fourteenth century. He invented series expansions for some trigonometric functions such as the sine, cosine and arctangent that were not known in Europe until after Newton. In modern terminology, these expansions are the Taylor series of the functions. Madhava gave an approximation of Pie as 3.14159265359, which goes far ahead of the four decimal places calculated by Aryabhata. Madhava's work with series expansions suggests that he either discovered elements of the differential calculus or nearly did so.

The foundation of Kerala school of astronomy and mathematics was attributed to Madhava. The school flourished between the 14th and 16th centuries and the original discoveries of the school seems to have ended with Narayana Bhattathiri (1559–1632 AD). In an attempt to solve astronomical problems, the Kerala school independently discovered a number of important mathematical concepts. Their most important results that include series expansion for trigonometric functions, were described in Sanskrit verse in a book by Neelakanta called Tantrasangraha, and again in a commentary on this work, called Tantrasangrahavakhya. The theorems were stated without proof, but proofs for the series for sine, cosine, and inverse tangent were provided a century later in the work Yuktibhasa (1530 AD), written by Jyesthadeva, and also in a commentary on Tantrasangraha. Their work, completed two centuries before the invention of Calculus in Europe, provided what is now considered the first example of a power series.

Ramanujan (1887-1920) is perhaps the most renowned of the modern Indian mathematicians. He made substantial contribution in mathematical analysis, number theory, infinite series and continued fractions. His contributions in number theory are very important and useful but his most enduring innovation may be the arithmetic theory of modular forms. In a significant paper published in 1916, he initiated the study of the Pie function. Ramanujan proved some properties of the function and speculated many more. As a result of his work, the modern arithmetic theory of modular forms, which occupies a central place in number theory and algebraic geometry, was developed by Hecke.

It is said that Ramanujan's discoveries are unusually rich and that there is often more to them than initially meets the eye. As a by-product, new directions of research were opened up. Examples of the most interesting of these formulae include the intriguing infinite series. One of his remarkable capabilities was the rapid solution for problems. The number 1729 is known as the Hardy-Ramanujan number after a famous tale of the British mathematician G. H. Hardy regarding a visit to the hospital to see Ramanujan. In Hardy's words, "I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable sign. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

The two different ways are

$$1729 = 1^3 + 12^3 = 9^3 + 10^3.$$

Generalizations of this idea have created the notion of "taxicab numbers". Coincidentally, 1729 is also a Carmichael number. He worked on divergent series. He sent 120 theorems on imply divisibility properties of the partition function. Partition of whole numbers is another similar problem that captured Ramanujan attention. Subsequently Ramanujan developed a formula for the partition of any number, which can be made to yield the necessary result by a series of successive approximation. Another milestone work by Ramanujan is known as The Ramanujan conjecture. In particular, the connection of this conjecture with conjectures of Andre Weil in algebraic geometry opened up new areas of research. The Ramanujan conjecture is an assertion on the size of the tau-function and proved many congruences for these numbers, such as $\tau(p) \equiv 1 + p^{11}\ (mod\ 691)$ for primes p which later on provided the basis for proving Fermat's Last Theorem. During his short life, Ramanujan independently compiled nearly 3,900 results many of which were completely novel but later on some are proved by other mathematicians. His original and highly unconventional results, such as the Ramanujan prime, the Ramanujan theta function, partition formulae and mock theta function, have opened entire new areas of work and inspired a vast amount of further research.

Dattathreya Ramchandra Kaprekar (1905-1988) made contributions towards various topics such as magic squares, recurring decimals, integers with special properties and much more. He is well known for "Kaprekar Constant" 6174 discovered in 1949. Take any four digit number in which all digits are not alike. Arrange its digits in descending order and subtract from it the number formed by arranging the digits in ascending order. If this process is repeated with remainders, ultimately number 6174 is obtained, which then generates itself. In general, when the operation converges, it does so in at most seven iterations. Another class of numbers Kaprekar described is the Kaprekar numbers. A Kaprekar number is a positive integer with the property that if it is squared, then its representation can be partitioned into two positive integer parts whose sum is equal to the original number (e.g. 45, since $45^2 = 2025$, and 20+25=45.). This operation, of taking the rightmost digits of a square, and adding it to the integer formed by the leftmost digits, is known as the Kaprekar operation.

Harish-Chandra (1923-1983) is perhaps the least known Indian mathematician and physicist who did his fundamental work in representation theory, especially harmonic analysis on semi simple Lie groups. His Collected Papers published in four volumes contain more than 2,000 pages. His style is known as meticulous and thorough and his published work is likely to treat the most general case at the very beginning. This is in contrast to many other mathematicians, whose published work tends to develop through special cases. Interestingly, the work of Harish-Chandra formed the basis of Langlands's theory of automorphic forms, which are a vast generalization of the modular forms considered by Ramanujan. He is also known for Harish-Chandra's c-function, Harish-Chandra's character formula, Harish-Chandra's regularity theorem, Harish-Chandra homomorphism and isomorphism, Harish-Chandra's Schwartz space and many more.

The most famous female Indian mathematician of all time, Shakuntala Devi (1929-2013), is commonly known as the "Human Computer". She was so called because of her incredible talent to do calculations without using any calculator. In Dallas, she even competed with a computer to give the cube root of 188138517 faster and she won. She went ahead to compete with UNIVAC, the world's fastest computer, to solve the 23rd root of a 201 digit number and she won that too! She was a woman with outstanding talent and outrageous world records and earned a place in 1982 edition of Guinness Book of World Records.

CONCLUSION

The present mathematical knowledge and development is not being achieved as a fruit from the sky, nor is a result of some magical tricks. These developed and finest facts and theories have been achieved by the continuous and effortless practices and researches of hundreds of mathematicians and historians for the centuries. Lots of people had contributed to the fruits, facilities and luxuries which we benefit from today. The contribution of Indian mathematicians is immense and extra-ordinary. From the concept of zero to the modern concept of computational number theory, their input is noteworthy. It is important to state that the outstanding contributions made by Indian mathematicians over many hundreds of years cannot be explained in few words or understood without being familiar to the field of mathematics. Indian mathematicians made immense contributions in developing arithmetic, algebra, geometry, infinite series expansions and calculus. Indian contributions in the field of mathematics influenced the world mathematicians when the Indian works got translated. It is the need of hour to promote ahead the heritage of great mathematicians so as to encourage and cherish the magnificent tradition of the country in mathematics.

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